Prove that $\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n-2)\times(3n+1)}=\frac{n}{3n+1}$ for all integers $n\geq 1$ **SCORE: _____ / 10 PTS**

by mathematical induction, showing all steps demonstrated in lecture.

BASIS STEP:

If $n=1$, $\boxed{\frac{1}{1\times 4}=\frac{1}{4} \qquad \frac{1}{3(1)+1}=\frac{1}{4}}$ 1

INDUCTIVE STEP:

1 $\boxed{\text{Assume } \frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3k-2)\times(3k+1)}=\frac{k}{3k+1}}$ $\boxed{\text{for some arbitrary but particular integer } k}$ 1

2 $\boxed{\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\underbrace{\frac{1}{(3k-2)\times(3k+1)}+\frac{1}{(3k+1)\times(3k+4)}}}$

1 $\boxed{=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}}$ FIRST & LAST 2 TERMS MUST BOTH BE SHOWN

$=\frac{k(3k+4)+1}{(3k+1)(3k+4)}$

1 $\boxed{=\frac{3k^2+4k+1}{(3k+1)(3k+4)}}$

1 $\boxed{=\frac{(3k+1)(k+1)}{(3k+1)(3k+4)}}$

$=\frac{k+1}{3k+4}$

1 $\boxed{=\frac{k+1}{3(k+1)+1}}$

1 $\boxed{\text{By mathematical induction, } \frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n-2)\times(3n+1)}=\frac{n}{3n+1} \text{ for all integers } n\geq 1}$

Evaluate $\sum_{k=1}^{300}(10k - 3k^2)$. **Your final answer must be a number (not involving arithmetic operations).** SCORE: _____ / 4 PTS

$$= 10\sum_{k=1}^{300} k - 3\sum_{k=1}^{300} k^2 \quad 1$$

$$= 10 \cdot \tfrac{1}{2}(300)(301) - 3 \cdot \tfrac{1}{6}(300)(301)(601) \quad 1 \quad 1$$

$$= -26683650 \quad 1$$

Find the 7^{th} term of $(11b-8g)^{26}$. **Your final coefficient may be in factored form as shown in lecture.** SCORE: _____ / 5 PTS

$_{26}C_6\,(11b)^{26-6}(-8g)^6$

$= \dfrac{26!}{6!\,20!}\,\underbrace{(11b)^{20}}_{}\,\underbrace{(-8g)^6}_{}$ $\frac{1}{2}$ $\frac{1}{2}$

PLUS $\frac{1}{2}$ IF YOUR FINAL ANSWER IS POSITIVE

$= \boxed{\dfrac{26\cdot \overset{5}{\cancel{25}}\cdot \cancel{24}\cdot 23\cdot \overset{11}{\cancel{22}}\cdot \overset{7}{\cancel{21}}\cdot \cancel{20!}}{\cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{20!}}}\,11^{20}\,8^6\,b^{20}g^6$ 1

$\frac{1}{2}$ $= \underbrace{26\cdot 5\cdot 23\cdot 11\cdot 7}_{}\cdot 11^{20}\cdot 8^6 b^{20}g^6 = 230230\cdot \underbrace{11^{20}}_{\frac{1}{2}}\cdot \underbrace{8^6}_{\frac{1}{2}}\,\underbrace{b^{20}g^6}_{1}$

If $f(x)=x^5$, expand and **completely simplify** the difference quotient $\dfrac{f(x+h)-f(x)}{h}$. SCORE: _____ / 5 PTS

3

$\dfrac{(x+h)^5-x^5}{h} = \dfrac{\cancel{x^5}+5x^4h+10x^3h^2+10x^2h^3+5xh^4+h^5-\cancel{x^5}}{h}$

$= \underbrace{5x^4+10x^3h+10x^2h^2+5xh^3+h^4}_{}$ 2

Expand and **completely simplify** the complex number $(3-2i)^4$. SCORE: _____ / 6 PTS

$1(3)^4(-2i)^0 + \underbrace{4(3)^3(-2i)^1}_{\frac{1}{2}} + \underbrace{6(3)^2(-2i)^2}_{\frac{1}{2}} + \underbrace{4(3)^1(-2i)^3}_{\frac{1}{2}} + 1(3)^0(-2i)^4$ $\frac{1}{2}$

$= \underbrace{81}_{\frac{1}{2}} + 4(27)(-2i) + 6(9)(-4) + 4(3)(8i) + \underbrace{16}_{\frac{1}{2}}$

$= 81 \underbrace{-216i}_{\frac{1}{2}} \underbrace{-216}_{\frac{1}{2}} \underbrace{+96i}_{1} + 16$

$= \underbrace{-119-120i}_{}$ $1\frac{1}{2}$